First Year Report on Case Study 4: Value Chain Optimisation

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1 Problem statement

Axxom AG defined a first case study in the area of value chain optimisation in the processing industries. The purpose of this case study was to provide the academic partners an example of a typical scheduling problem in the processing industries which does not require in depth understanding of production processes and the complexity of which is suitable for the testing of different modelling and optimisation strategies.

The chosen example is the production of lacquers for the car industry in a pipeless plant. In the first version of the case study which was presented at the meeting in Grenoble in April 2002 and was subsequently refined and presented in more detail at the following meetings, three different types of lacquers are produced. The production of one batch of lacquer requires 5-7 steps, including laboratory testing and a correction step. In the plant, 8 production resources and 6 mobile vessels have to be considered. Some resources and one vessel are only used for one type of recipe while the others can be assigned freely. Operations cannot be interrupted (non-preemptive scheduling) and the material remains in the mixing vessels while several steps, in particular laboratory testing and eventually additional mixing, are performed. This leads to a situation where the operation times of the vessels are variable because they result from the scheduling of the operations for this batch. For some operations, more than one piece of equipment can be used. In the original problem, the amounts defined by the production orders can surpass the capacities of the vessels, so a splitting step has to be included. However, this has not yet been considered in the modelling efforts. Each production order has a release date (earliest starting time) and a due date (deadline). The main interest is in meeting the deadlines.

The problem belongs to the class of job shop problems with the additional complication of the assignment of the mixing vessels as a second resource required to perform the operations on some machines. A second feature which is non-standard in job shop problems but commonly found in chemical plan scheduling problems is the presence of constraints on the storage times of some products. These constraints are expressed as bounds on the differences between the starting and the ending times of two operations of a job. Three types of such constraints occur: start-start, end-start, and end-end constraints.

The time horizon of the entire problem defined by the earliest release date and the latest deadline of all jobs comprises approx. 7 weeks, the processing times on the machines range from few hours up to three days for laboratory testing.

2 Modelling efforts

Two different modelling approaches have been pursued in the first year of the project: modelling as a mixed-integer linear program (MILP) in order to apply commercial tools for the solution of mixed-integer linear optimisation problems, and modelling by timed automata in order to apply variants of reachability analysis to scheduling problems.

2.1 Modelling as MILP[3]

The standard approach to modelling of chemical production scheduling problems is the STN (state-task-network) formulation [5][10][4] with a discrete time representation where changes of the states of the model can only occur at equally spaced time instants. As any possible event can occur at any of these instants, which is modelled by a binary variable, these models quickly become very large if the number of events is large, i. e. the scheduling horizon is large compared to the maximal resolution required to meet the constraints which is implied by the operation times. In this example, this ratio becomes rather large and hence such models are not suitable here.

An alternative is the use of a continuous representation of time where the time at which an event (e. g. the start or the end of an operation) occurs is a real variable. The number of such events can be computed from the production orders and the recipes (plus eventually a worst-case estimate for sequence dependent operations such as cleaning operations). In the example considered here, there a typically not more than a few hundred events whereas a discrete time STN formulation leads to several hundred thousand binary variables. On the other hand, though, the relative positions of the events must then be represented by binary variables in order to express constraints of the type that two operations cannot be performed on the same machine at the same time but either operation A must finish before B starts or B must finish before A starts.

It turned out, that a standard formulation of a continuous time MILP model also gives rise to rather large models with thousands of binary variables for problems with more than 10 jobs. Therefore the model is reduced by additional heuristics. These heuristics state that operations of jobs where the time window does not overlap have to be performed on a machine in this order, and that jobs of the same type may not overtake each other. Both heuristics do not affect the performance and exclude alternatives which cannot lead to an improvement of the schedule. Such heuristics eliminate a considerable number of binary variables, and as a result the number of binary variables here only increases linearly with the problem size. In order to speed up the solution of the optimisation problem, in a first stage an additional heuristics is included where the operations on one machine are always ordered according to the EDD rule (earliest due date first). After a feasible solution is found with this restriction, the unrestricted problem is solved, using the first result as an upper bound in the branch-and-bound strategy. GAMS [1] was used as a modelling language, and CPLEX [2] as the MILP solver.

First tests of the performance for problems of increasing size showed that problems with up to 10 jobs can be solved very efficiently. The solutions are either optimal or have small integrality gaps. In these cases, the removal of the EDD heuristics usually does not lead to an improvement of the solution. For more jobs the computation time increases sharply and exceeds 40 minutes on a high end PC with 1 GB of memory for 12 and more orders. Nonetheless, the obtained solutions are delay-free for up to 20 jobs. Nonzero values of the cost function are caused by early termination of jobs and by a small tie-breaking term which causes a bias towards scheduling the tasks within a job as late as possible. The second stage of the optimisation did not lead to an improvement of the solution for larger problems.

In order to cope with the exponential growing complexity, a decomposition strategy based on a moving horizon approach has also been implemented. The idea is to order the jobs according to their release dates or due dates and to optimise the scheduling of the next jobs, to fix the first job in the resulting schedule (at least with respect to the corresponding binary variables) and to iterate with one new job replacing the fixed one. Preliminary investigations show a reasonable behaviour of this algorithm with small gaps in each individual step for most sub-problems.

Future work will concern the investigation of alternative model formulations, e. g. not assigning individual vessels but restricting the number of vessels that are used simultaneously, and on a more detailed elaboration and comparison of moving horizon strategies.

2.2 Modelling as TA

Timed automata provide a flexible tool to model scheduling problems in a modular fashion. Considerable effort has been invested in developing a detailed UPPAAL model of the case study [6]. The description of the problem delivered by AXXOM turned out to be partly ambiguous for scientists not familiar with the underlying problem. The automatic transformation of the description used in AXXOM's tool ORION PI to a format that can be used in model checkers is a possible area of future work. A TA model has also been developed at VERIMAG. In both models, timed automata are defined for the jobs which are synchronised with simple resource automata (counters) that represent groups of interchangeable resources (i. e. similar resources are not modelled individually).

In [7] the TA model is complemented by mechanisms that implement scheduling rules to reduce the size of the state space. Three different rules are used:

- avoid lazy executions, i. e. schedules with useless waiting times

- avoid overtaking by jobs with later due dates of the same type (for the goal of minimising tardiness, this cannot improve a schedule because the simple interchange of the two jobs would always lead to a schedule with a better or the same value of tardiness)
- introduce minimal separation times to avoid constrain violations caused by the requirement of non-laziness.

With these additional rules, an optimal schedule in which all due dates are met could be found for the full problem with the tool IF [8].

2.3 First conclusions and future work

The MILP formulation on the one hand is very general, in particular all sorts of constraints among jobs and subtasks and arbitrary piecewise linear cost functions can be formulated. The solution procedure provides an optimal solution, or a solution and a lower bound on the cost function which indicates the maximal potential for improvement. On the other hand, the formulation of the model is cumbersome and tedious, and the solution times may become prohibitive for larger models. A general experience is that the application of generic modelling methods (as STN) to realistic problems leads to models which are difficult to solve, so problem-specific models are recommended, thus increasing the modelling effort.

TA models on the other hand are graphical and modular, and hence more transparent. After the addition of rules which reduce the size of the state space, feasible solutions can be found with the available tools. Optimisation for arbitrary cost functions seems difficult.

The issue of batch splitting and batch size optimisation has yet to be studied using both techniques.

The example studied in the first year is a simplified version of the real problem of lacquer production, and real value chain optimisation problems go far beyond the scheduling of a single production site. Therefore, during the second year of the project, more complex variants of the case study will be provided by AXXOM and studied by the other partners in the project. Possible extensions of the model are:

1. increasing scale: more jobs, more resources, more product types
2. increasing difficulty: more constraints, realistic cost functions, sequence dependent durations
3. consideration of stochastic elements: varying operation times, random need for re-work embedding in a realistic value chain optimisation problem.

Another issue that will be pursued is the combination of MILP optimisation techniques with TA models in order to combine transparent modular modelling with the solution power of MILP algorithms. A first step in this direction is reported in [9].

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